Analyzing Math Word Problems with Digital Video: A Usage-Based Approach

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Abstract

Math word problems cross semiotic boundaries and create challenging translations from spoken or written language to the language of mathematics (Lemke, 2003; Radford, 2003). The theoretical approach to the data for this paper is derived from Michael Tomasello's (2003) usage-based theory of language acquisition, along with ideas about learning and language acquisition related to the work of Russian Psychologist Lev Vygotsky (Scribner, 1997; van Lier, 2004; Vygotsky, 1978; Wertsch,1998). This is a descriptive, interpretive case study that presents digital video data, visuals, and transcriptions of three cases of college-level students solving the same problem. The main objective is to understand *semiosis* as students create meaning during different steps leading up to the creation of the video, including a close analysis of the video with speech, visuals, and acts of pointing as the units of analysis. The findings include a multi-modal presentation of how students comprehend the word problem and work toward a solution. Overall, this paper provides an option to improve students' metacognition and self-regulation, specifically in the process of moving from one semiotic system to another. The theoretical framework and procedures can also be applied to word problems in other STEM disciplines.

Introduction

One of the well-known challenges in teaching adults to read and write academic English is how to prompt some deep level of engagement with language, content, and critical thinking. In other words, how to teach students to use writing as a way of thinking, in addition to reading for ideas and solutions for different situations and problems. These same kinds of broad literacy and critical thinking issues exist in developmental math courses, and in one way or another, these broad issues of learning to read and write critically exist across all academic disciplines and applied technologies (Fang & Shleppegrell, 2010).

Math word problems often cause difficulties for students even when the mathematical techniques involved in solving these problems are well known. The first apparent reason for this is that students do not understand what the problem is about. Specifically, they cannot determine what is known and what has to be found (Korpershoek, Kuyper, & van der Werf, 2015; Lemke, 2003). The next difficulty is in translating the problem, formulated in English, into the language of mathematics, along with understanding what meaning is interpreted by participants into the variables, constants, equations, inequalities, and their solutions from the original problem. According to Lemke (2003), there is a growing understanding that “natural language, mathematics, and visual representations form a single unified system of meaning-making” (p. 215). (It should be noted here that by using the term “natural language,” Lemke is using “natural” as a term to distinguish any standard language as different from the language of math). Thus, practice solving math word problems can help students to attain a deeper understanding of the English language, the language of mathematics, and develop metacognitive abilities. (Goos, Galbraith, & Renshaw, 2002; Koepershoek et al, 2015; Sáenz-Ludlow, 2006).

The main purpose of this paper is to inform instruction and share this process and data with other teachers to adapt as needed. The main question we are working with at this stage of the research is *what do the differences in the visuals and the discourse illustrate about the participants' interpretations of math word problems and their metacognition?*

We first begin with the overall synthesis of different approaches to set up and analyze the digital video activities. This will be followed by a review of relevant literature and methods. The results and conclusions will be followed by limitations of the study, implications for the classroom, and avenues for further research

Theoretical Approach

Signs, Semiotics, Signification, and Mediation

A sign is something that stands for “something else” (Eco, 1976, p. 16); the sign gains meaning through social interaction (Eco, 1976; Peirce, 1991; see also van Leeuwen, 2005). Signification is the process of creating a sign; that is, assigning meaning to an object, the self, and the world (Vygotsky, 1978; van Lier, 2004). What is signified simultaneously begins to *mediate* meaning, most often goal-oriented and intentional.

Mediation refers to the way humans use concrete and abstract tools, including sign systems (e.g. language), to interpret, plan, regulate, and generally make meaning. A well-known example includes tying a knot to remember something or count something. Moreover, the examples from Tomasello (2003), which are presented below, illustrate how interlocutors use the surrounding context and language to signify and mediate meaning.

A related term is *semiosis,* which is the development of signs into other signs and into systems; the constant recreation and interpretation of a sign (van Lier, 2004, p. 113). An example of the contextual, social, and local nature of signification and semiosis is the intended meaning and contextual interpretation of a yellow light. A green light officially signifies go; a red, stop, and a yellow light's intended legal meaning is to slow down and stop if it is safe to do so. However, it is very different to slow down and try to stop at a yellow light in a big city where densely packed rush-hour traffic is zooming along at forty-five miles per hour, rather than in a small town where a few cars are traveling at thirty-five miles per hour. The point is that the lights mediate the movement of traffic, and the lights' meaning is embedded in a specific social context.

Tomasello's Usage-Based Approach

There are many conflicting and complementary theories and approaches to the teaching and researching of reading, writing, cognition, and first and second language acquisition, including theories related to Vygotsky (Holme, 2010; Thorn, 2005; see also Wilson, 2006). The first author has been working to adapt Tomasello's (2003) *usage-based* approach to EAP courses and Freshmen composition courses for several years (Unger, 2018). Tomasello's theory of language acquisition is contrary to Noam Chomsky's Universal Grammar. Tomasello's work also stands out among other approaches to cognition and language (Atkinson, 2014; Langacker, 2005; Wilson 2006). Moreover, the current paper is an effort to cross academic borders and bring a theory of language acquisition into STEM courses; for this paper, mathematics.

Overall, Tomasello argues that language acquisition is grounded in the unique cognitive characteristics of humans and the way they socially interact (Ibbotson & Tomasello, 2016; Tomasello, 2003). Specifically, the ability to read the intentions of others is prominent, as well as recognizing others as having intentions they can direct toward objects, the intentional states of others, and the world around them.

To paraphrase this idea of the wide-ranging nature of intentions, suppose I wanted to have you meet with Bob at the car mechanics tomorrow and pick up my wife's Toyota, and check the invoice for any price over a certain amount. In this way, my intentional meaning causes a number of other events to occur.

An author of a math word problem usually has a specific outcome in mind, or even if the answer is more open to interpretation, understanding the author's intention is crucial when trying to translate a word problem into a mathematical equation. The same types of students' understandings of author intentions for math word problems also apply to physics problems, chemistry problems or any number of different problem-solving situations (Korpershoek et al., 2015). All of these examples of intention-reading, along with shared attention and shared understanding between two or more participants, create a communicative/socio-cognitive event, which Tomasello (2003) calls a *joint attentional frame.*

Three examples, paraphrased from Tomasello (2003), illustrate the main features of Tomasello's joint attentional frame. The examples also demonstrate the applicability of Tomasello's ideas for second language acquisition (SLA) contexts, as well as a broad number of academic disciplines and applied technologies, including math. Tomasello proposes that a natural part of human interaction is shared attention through context and some manner of directing the attention of one or more *others*.

One example is an adult with a diaper in her hand walking into a room with a baby. The adult looks at the diaper; the baby follows the adult's gaze to the diaper, and the baby understands the adult's intentions and the sequence of events that will follow. Perhaps later the adult will return to the room with a stuffed rabbit and the same triadic kind of situation will occur: the baby will look at the stuffed rabbit as the adult might say "Are you ready to play with your rabbit?" Through this triadic arrangement, similar to when the adult came into the room before with a diaper, the baby and the adult both know the intended sequence of events that will follow, which involve playing with the stuffed rabbit. Most important, the baby is engaged in language learning.

A third example applies to more common social interactions. To paraphrase Tomasello's (2003, p. 25) example of how a joint attentional frame might occur in a more typical communicative context, suppose a traveler who only speaks English stands in the middle of a bus station in Yemen. A man walks up to the traveler and begins speaking Arabic. Suppose the man is asking about directions for the next bus to another town. The traveler will not understand what the Yemeni man is asking. Now suppose the interaction happened next to the ticket booth where there was a clock and a schedule of all arriving and departing busses, and perhaps a map was on the wall with bus routes. The Yemeni man can point at the clock, the map, and the schedule on the wall and ask the questions again (this pointing does not have to be a hand; can be the eyes, a minor gesture, or other means to draw attention to the related object or topic). This time, although the Yemeni man will still not be completely understood by the traveler, his intended-meaning becomes stronger. By speaking, pointing, and using the visuals available in the context, the Yemeni man has created a joint attentional frame similar to the earlier examples of the baby and the adult, though using more complex signs.

A Shared Attentional Frame

From classroom applications of Tomasello's work, the first author has developed a model of a *Shared Attentional Frame* (Unger, 2018), with speech, a visual, the act of pointing, and a *third* space of intended and interpreted meaning (See Figure 1). This model has evolved from Tomasello's (2003) rendering of the "Structure of linguistic symbol" (p. 29) and a "Joint Attentional Frame" (p. 28), in addition to ideas about interaction, second language acquisition, and learning and development from McCafferty (2002), Wertsch (1998) and Vygotsky (1978). In one way or another, each of these authors highlight the importance of recognizing the embodied nature of language and the dynamic nature of sign creation and use.

The model in Figure 1 suggests how speech, visuals, and acts of pointing create a dynamic third space of meaning where intended meaning is created and interpreted (the term is not related to Gutiérrez's 2008, conceptualizing of a "third space." The concept of a third space in this research was derived differently and independently). The interpretation of speaker/author intentions is signified by variance in the black arrows pointing in both directions. Although the model unavoidably appears static, the activity depicted in the model is dynamic, fluid, and dialogic (Bakhtin, 1986, 1981; Clark & Holquist, 1984).

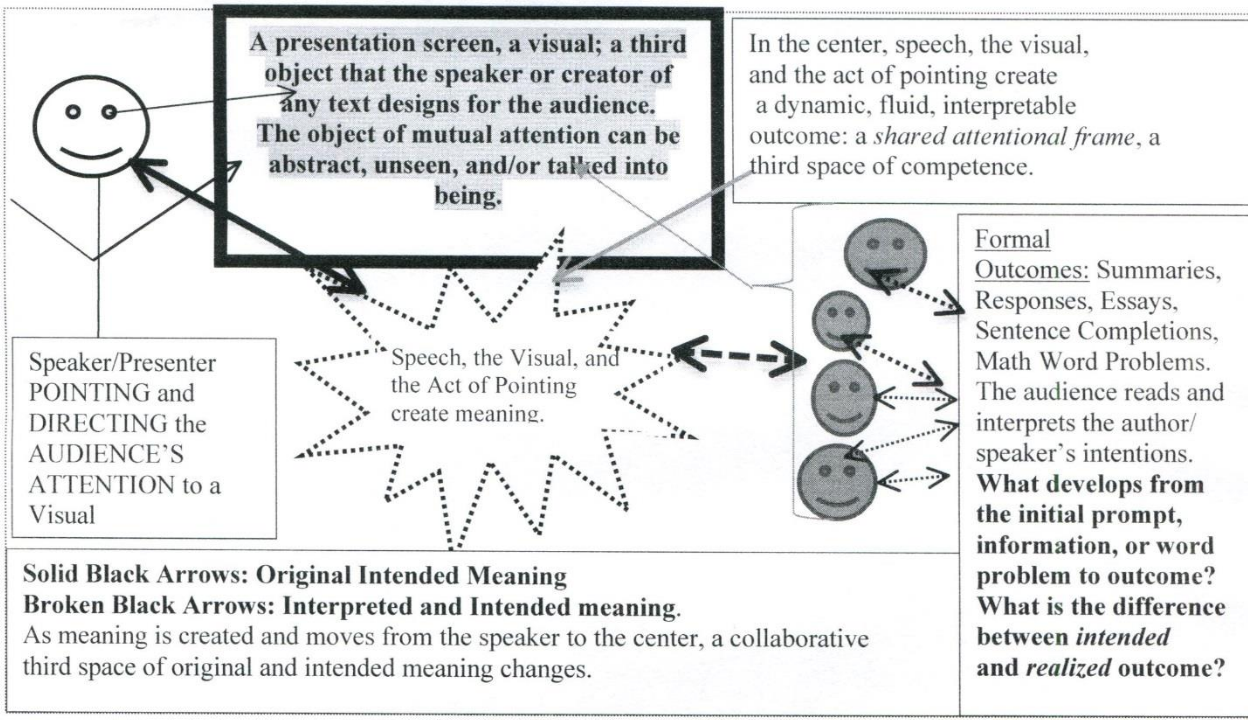


Figure 1. Model of a Shared Attentional Frame (not to be read as a static display).

To summarize the overall theoretical perspective, signification, mediation, and the constant development of sign reading, using, and producing abilities is central to human activity. These sign-making and sign-using abilities are related to higher level abstract reasoning (Scribner, 1997). An important part of this process of semiotic mediation is the way humans read each other's intentions (Tomasello, 2003).

Related Literature

One major challenge with doing a literature review with this type of an interdisciplinary study is the range of research on metacognition, self-regulation, math word problems, and the complex relationship of metacognition and self-regulation (Bene, 2014; Goos, Galbraith, and Renshaw, 2002; Schraw, 2007; Ziegler, 2014). Various perspectives on self-regulated learning and metacognition are often epistemologically contrary to Vygotskian and semiotic approaches to learning and development. Approaches associated with cognitive science and educational psychology (e.g., Hsu et al., 2016; Shraw, 2007; Sperling et al., 2004) tend to categorize different features of cognition with more defined boundaries and parameters. In contrast, the Tomasello, semiotic, and Vygotskian based approaches used in this paper focus on social interaction with signs, most importantly, language.

Metacognition

A tangential objective of this paper is to unpack the concept of *metacognition* in the context of solving math word problems. Metacognition is generally understood as thinking about one's own thinking (Hsu, Iannone, She, Hadwin, & Yore, 2016; see also Schraw, 2007). Metacognition is also considered a part of *self-regulation* (Sperling, Howard, Staley, & Dubois, 2004; Ziegler, 2014); however, from our perspective, the fine line between metacognition and self-regulation disappears due to the semiotic nature of the overall approach to the data. From the literature on metacognition and self-regulation (Sperling et al., 2004; Hsu, et al., 2016; Schraw, 2007), [1] metacognition is generally understood as "knowledge of cognition" and "regulation of cognition" (Sperling et al., p. 118).

Goos et al. (2002) described metacognition as "students' awareness of their own cognitive processes and the regulation of these processes in order to achieve a particular goal" (p. 193). Goos et al. used Vygotsky's Zone of Proximal Development and discourse analysis as a foundation for their research with high school seniors who collaboratively solved word problems. Regulation is described as "planning an overall course of action, selecting specific strategies, monitoring progress, assessing results, and revising plans and strategies if necessary" (p. 193). In addition, Goos et al. describes *metacognitive acts* as, "where new information was recognized or an assessment of particular aspects of a solution was made" (p. 199).

Metacognitive acts described in Goos et al. were made explicit during group collaboration and conversation. We are borrowing the construct of metacognitive acts as a way to identify contrasting moments in the data where participants are adjusting their path from problem to solution.

Reciprocal Teaching

Participants were first provided with the math word problem on a regular 8.5 by 11inch sheet of paper. They were told to first make a visual sketch of the problem and any calculation they needed; then told they would be putting this information on a larger sheet of poster paper and video-record an explanation of their solution as if they were teaching an imagined audience how to solve the problem. Participants were told that teaching how to solve the problem was related to the concept of *reciprocal teaching* (Palinscar & Brown, 1984). Although the concept was broadly applied to the participants as having a student take on the role of teacher, the original construct is more specific. For reading comprehension, Palinscar and Brown (1984) focused on "*summarizing*

[1] It should be noted here that we acknowledge that for this study we are bringing together several distinct frameworks in an effort to build an interdisciplinary approach to word problems in general and math word problems specifically.

(self-review), *questioning, clarifying,* and *predicting"* (p. 120, italics in original). Reciprocal teaching has also been used with some success with math word problems and for math peer-tutoring (Meyer, 2014; Dufrene et al., 2009)

Math Word Problems, Reading, Videos, and Visual Displays

In a review of the literature on reading and math word problems, along with physics and chemistry problems, Korpershoek et al. (2015), found that most studies focused on the younger, elementary school students, and on students with some kind of learning disability (see also Zhang & Xin, 2012). In our own review of the literature, we found the same. We also did not find any case study research and discourse analysis on adult students' math word-problem strategies that used Tomasello, Vygotskian, and semiotic perspectives, along with multi-modal steps in creating the visuals leading to a video explanation. The closest perspective to ours was from Goos et al. (2002), whose research became important for our understanding of metacognition. Furthermore, Korpershoek et al (2015) and Sáenz-Ludlow (2006) are relevant for their semiotic approaches to word problems. Additionally, we did not find any math word-problem research that used speech, visuals, and acts of pointing as units of analysis. However, what we did find were perspectives on word problems that consistently and unavoidably brought up general literacy and semiotics. Also, the use of visual representations, including multimedia representations of math word problems, seemed prominent in the research (Casey, 2003; Oldknow, 2009).

One of the more relevant studies for our research was a study of 1,446 Dutch students at the secondary education level, with a large percentage of immigrants included in the study (Kopershoek et al.2015). The authors emphasized the semiotic nature of word problems in math, chemistry, and physics, and how reading comprehension was also related to success with word problems. However, they did not find any relationship between ethnicity, gender, and success with word problems. They also found that math ability translated to better scores on examinations in physics and chemistry. Kopershoek et al. suggested that reading comprehension should be taught along with word problems in math, as well as teaching reading in chemistry and physics classes.

Sáenz-Ludlow (2006) also took a semiotic perspective on math word problems (see also Sáenz-Ludlow & Presmeg, 2006). Participants in this study were elementary-aged students. Sáenz-Ludlow (2006) stated that "the learning of mathematics entails both the interpretation of mathematical signs and the construction of mathematical meanings through communication with others" (p. 183). Following Wittgenstein (1991) and Freires' (1970/2001) positions on language play (as cited in Sáenz-Ludlow), Sáenz-Ludlow (2006), studied *language games* and *interpreting games* in the math classroom. As stated in her paper "In general, *language games* can be considered as essential tools for communicating while *interpreting games* can be considered essential tools for *teaching and learning*" (200).

The interpretation game emphasized interaction between students and teachers as they went through a "dialogical interaction" (p. 203) about a specific interpretation (see also Bakhtin, 1986; Wertsch, 1998). For example, the students and teacher would have collaborative conversations about the meanings of mathematical signs in simple equations, like the equal sign (=). The data presented and analyzed as interpreting games were transcribed recordings of the teacher and students negotiating the meaning of what the signs meant in equations. Sáenz-Ludlow (2006) exposed the power of communication and interpretation for these students to better understand the way they were reading and interpreting signs. Video transcriptions and a semiotic approach were essential to her findings.

Recall that Goos et al., which informed our understanding of metacognition, used classroom observation and audio and video data of small groups of secondary students solving math word problems. Goos et al., emphasized Vygotsky's Zone of Proximal Development (ZPD) and social interaction in their study. The ZPD is understood broadly as collaboration with others, particularly more capable others, and increases the chance of working on problems at a higher level of proficiency than one can do alone (Vygotsky, 1978; see also Gutiérrez, 2008, for an informative discussion of the ZPD).

The word problems in Goos et al. included compound interest, projectile motion, and Hooke's Law. Collaboration was positioned as mediating metacognition. A major finding was that students' willingness to actively discuss and engage in each other's interpretation was related to improved metacognition.

Video or other types of digital tools are also prominent in studies on the math meaning-making process and the improvement of outcomes (Casey, 2003; Green & Maushak, 2014; Lantz-Andersson, Linderoth, Säljö, 2009; Oldknow, 2009). Most important for our current study is how any type of digital tool transforms the meaning making process (Lantz-Andersson, Linderoth, Säljö, 2009; see also Wertsch,1998). Also prominent in the literature on math word problems, including other STEM fields, is the transformative nature of tool-use and the power of visual representations. (Múñez, Orrantia, &Rosales, 2013; Shanahan, Shanahan, & Misischia, 2011).

All of the studies reviewed, in one way or another, point toward the positive influence of contextualizing math word problems. The trend in the research is to discover and enhance students' ways of thinking about word problems. Another common thread is the positive relationship between literacy and math word problems, and the need to support literacy as a normal part of teaching and learning about math word problems (Greeberg, Ginzburg, & Wrigley, 2017; Kong & Orosco, 2016).

Method

The General Research Design

At this initial stage of the data analysis, the emphasis is on the English language; not on mathematics. We are using a multiple case design (Yin, 2003) with three units of analysis for each case: speech, the visual, and the act of pointing, triangulated to identify *metacognitive acts* and other moments of discourse that reveal how participants are interpreting and solving the word problem. A precise transcription of the act of pointing is beyond the scope of our emphasis on metacognitive acts found in the data. The act of pointing, as it is synchronized or not with speech, is broadly defined. The act of pointing is an important issue to be taken up in further research. The data analysis procedures involve contrasting the descriptions of the cases with one another.

The Participants as Cases and the Context for the Study

The data were collected at a four-year regional college in the southern U.S. The general population could be described as a diverse four-year college population, and the standards for admissions were appropriately open-access to provide opportunity to many students who otherwise might not attend college. From this population and for the first data collection period (other data with variations from these procedures are still undergoing analysis), we recruited a small set of participants for three different math problems. In order to maintain anonymity, we are only providing a minimal amount of background data on the participants.

The data from the three cases were collected outside of the classroom and were part of an initial nine cases from the Spring semester of 2015. These were randomly recruited, paid cases; that is, the first author distributed flyers around the campus. The flyers offered students twenty-dollars for spending an hour of their time to make a video explaining their process for solving the word problem, and answering questions about their videos, along with a few background questions. This protocol has since been adjusted through several semesters of data collection from developmental math classes. These three cases have served as reference data for deciding on different strategies to apply to other groups, which will be discussed in forthcoming papers.

Microgenesis

Because we are using digital video, which captures the cycle of *signification* and *mediation* as a semiotic system of *speech,* the *visual*, and the *act of pointing*, the Vygotskian socio-historical concept of how development takes place as a *semiosis* over short periods of time, a *microgenesis*, is important for identifying *metacognitive acts* in the data. According to Vygotsky (1978), “to study something historically means to study it in the process of change” (p. 65). Furthermore, he proposed “it is only in movement that a body shows what it is” (p. 65).

Microgenesis frames development as unfolding before one's eyes over short periods of time; as short as seconds or minutes, or longer periods (Wells, 1999). Lantolf (2000) described the microgenetic domain as follows: "Where interest is in the reorganization and development of mediation over a relatively short span of time (for example, being trained to criteria at the outset of a lab experiment; learning a word, sound, or grammatical feature of a language)" (p. 3). Wertsch (1985) provided definitions for two types of microgenesis:

> The first type of microgenesis identified by Vygotsky concerns the short-term formation of a psychological process. The study of this domain requires observations of subjects repeated trials in a task setting. . . The second type of microgenesis is the unfolding of an individual perceptual or conceptual act, often for the course of milliseconds (p.55).

The data from the three cases display unique microgenesis across different steps and modalities, from the initial problem to final solution, as participants are prompted into a shared attentional frame to explain their answers.

Results

To provide context for the results, the research question is restated: *what do the differences in the visuals and the discourse illustrate about the participants' interpretations of math word problems and their metacognition?*

The Three Cases

To provide a clear description and interpretation of the data, we will juxtapose the different steps in the process from the initial calculations on the eight-and-a-half inch by eleven-inch typing paper to the depictions on the poster paper, including transcribed discourse and comments made by participants about their videos and history with mathematics.

Recall that we are looking for where and when metacognitive acts occur in the participant's problem-solving process, with an emphasis on the video. Also recall that a metacognitive act is "where new information was recognized or an assessment of particular aspects of a solution were made" (Goo et al. p. 199). Also recall that these metacognitive acts developed as a shared attentional frame, and we are using speech and the visual as our main units of analysis, with the act of pointing providing a background unit of analysis for triangulation (Yin, 2003).

As mentioned earlier, all participants were given the same general directions to assume the role of teacher; that is, act as if they were teaching the solution to someone else (i.e. a kind of reciprocal-teaching task). They were shown an example of the first author bungling a simple gas-mileage calculation (see 2015 Digital Model of a Word Problem password rabbit15), and it was emphasized that the visual and pointing at the visual, along with their explanation, would be the three most important features of their recorded explanation.

With the overall nine cases, generally, participants took about fifteen minutes doing the initial calculation, approximately another fifteen minutes making the poster, and about two or three minutes of actual recording. Very brief pre- and post-interviews also took place, which sometimes extended to include general topics about school. These usually took another fifteen minutes, including the time for introductions, small talk, and time to watch the videos. Altogether, it seemed to take just under an hour to complete the data collection for each participant.

The Three Participants: General Background

Case Margo

At the time of the data collection, Margo was about to graduate with a B.S. in Environmental Policy; she was a guest student from another state taking courses at the College, and she was heading to graduate school, though we did not discuss where. She expected that math would be a challenge in her future studies. Margo said that she had been intimidated by math most of her life, but she thought she had made progress lately. She had previously failed pre-calculus and waited until the very end of her B.S. degree program to take pre-calculus and passed the second time.

Case Sandra

At the time of the data collection, Sandra was a freshman in a developmental math course. When she came in to make her video, she was with two other friends who also made videos on different word problems. In the post-video interview, Sandra said she wanted to major in business. She said she has always had trouble with math and had particular difficulty taking tests: "I'm not good at it." Sandra said she misunderstood one kind of problem for another (e.g. "I take a linear equation as something else"). After another participant in this group was talking about "basic math," Sandra said, "basic math, we can understand it, but when we come to college, the math is like different." When the first author asked about specifics, all three talked at once, and it was hard to hear. Sandra can be heard saying "the variables moving in and that's confusing." The first author asked if they were referring to Algebra and Calculus, and all can be heard mumbling affirmatively to this leading question. The specifics of Sandra's perceptions of what is different from high school and college math are still unknown and might be a productive path of further inquiry with other students.

Case Isabella

Isabella was the most advanced of the three, which is understandable because at the time of the data collection, she was a math tutor. She was also in her junior year and was majoring in Cell Biology and Biotechnology. She said "I love math," and she reported positive experiences in her K-12 schooling with math. In the post-video interview, she described the question as "straight up."

Three Initial Responses to the Brick Problem

As mentioned previously, participants were given a sheet of paper with the question and asked to do an initial sketch underneath the problem. They were reminded to sketch out the problem as if they were teaching how to solve the problem for someone else (see also The Brick Problem; password otter17).

Interpretations of the Initial Calculations

Case Margo's initial response on the letter-sized paper showed darker numbers that were added with one of the marker pens used to make the poster (see Figure 2 below). The second author interpreted this as guessing numbers, which he said was a "legitimate approach when nothing better comes to mind." Setting the brick on the left side of the representation and assigning one pound to the brick was described by the second author as a "wrong choice, a guess." When she moves to the poster (Table 1), she strips away the guesses and equations in black. Another noticeable feature of the initial calculation is the conceptualization of a scale, with two sides that need to be balanced.

Case Sandra's Initial Calculation of the Word Problem

Case Sandra' initial diagram and response (See Figure 3 below) emphasizes the brick rather than conceptualizing a scale or balance between two sides. She also provides directions in words and sentences on both the initial calculation and poster for the video. The second author pointed out that Sandra's first mistake was deciding that one brick is one pound. Additionally, Sandra should have been multiplying instead of adding. Overall, the way she conceptualized the problem as the brick alone, without a scale, along with assigning the one brick to be one pound, suggests that Sandra is misunderstanding the basic premise of the problem.

Case Isabella's Initial Response to the Word Problem

On her initial calculation (See Figure 4 below), Isabella knew exactly how to set up the equation with a variable. She presents the conceptualization of the scale with the formula embedded inside the scale, transforming the wording of the problem and the necessary variables into one image, a balanced scale. Nothing seems to be erased at all. The poster for her video expanded on the initial calculations and becomes more oriented toward instruction (see Table One).

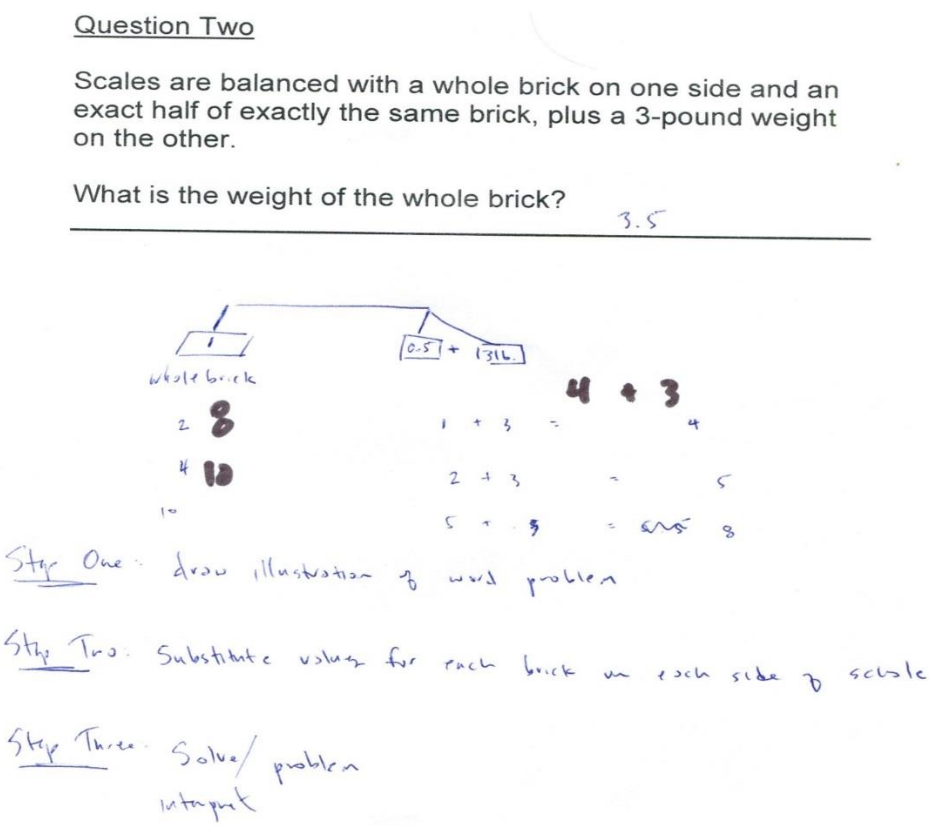
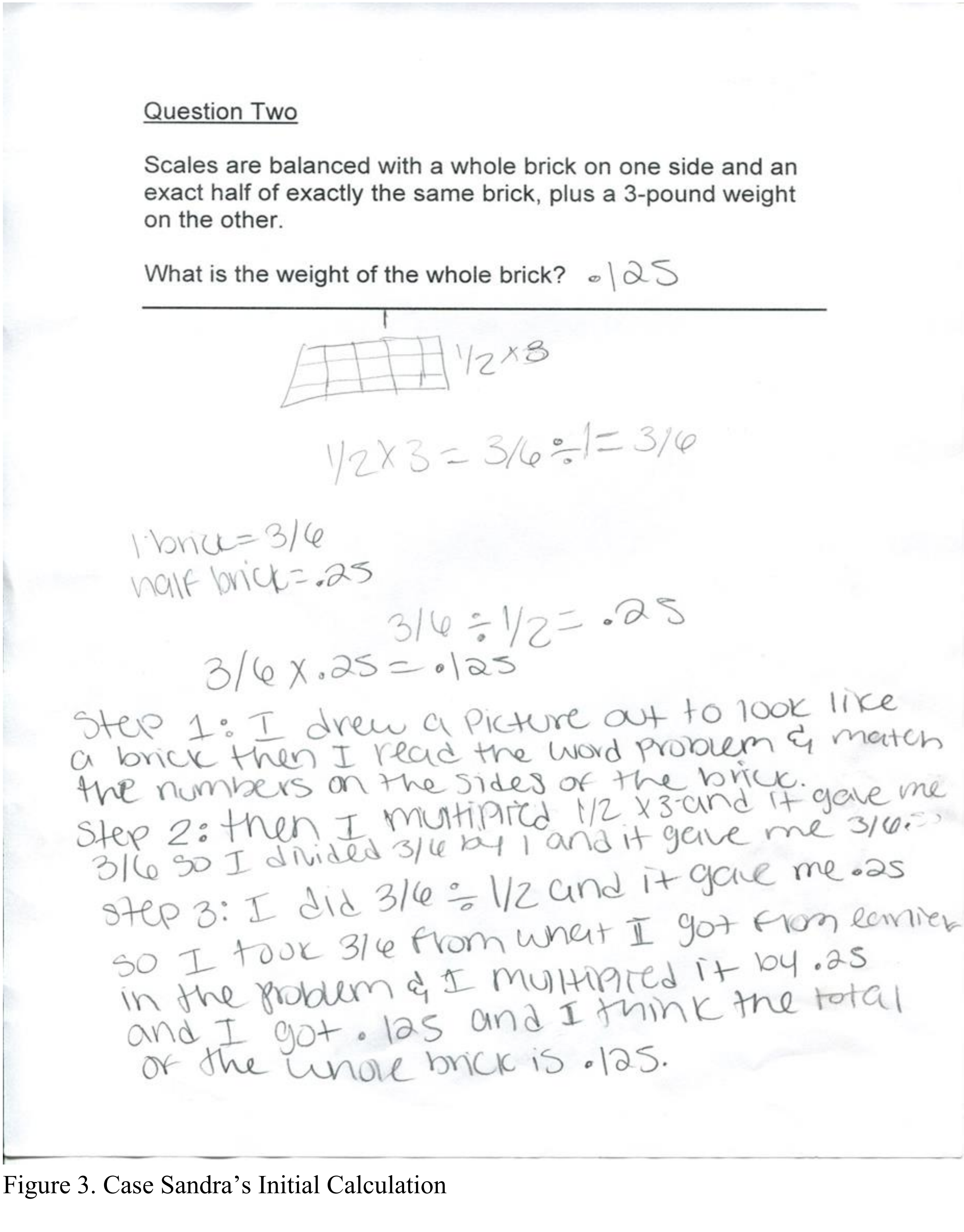


Figure 2. Case Margo's Initial Calculation



Question Two

Scales are balanced with a whole brick on one side and an exact half of exactly the same brick, plus a 3-pound weight on the other.

What is the weight of the whole brick? .125

1/2 x 8

1/2 x 3 = 3/6 ÷ 1 = 3/6

1 brick = 3/6
half brick = .25

3/6 ÷ 1/2 = .25
3/6 x .25 = .125

Step 1: I drew a picture out to look like a brick then I read the word problem & match the numbers on the sides of the brick.
Step 2: then I multiplied 1/2 x 3 and it gave me 3/6 so I divided 3/6 by 1 and it gave me 3/6.
Step 3: I did 3/6 ÷ 1/2 and it gave me .25 so I took 3/6 from what I got from earlier in the problem & I multiplied it by .25 and I got .125 and I think the total of the whole brick is .125.

Figure 3. Case Sandra's Initial Calculation

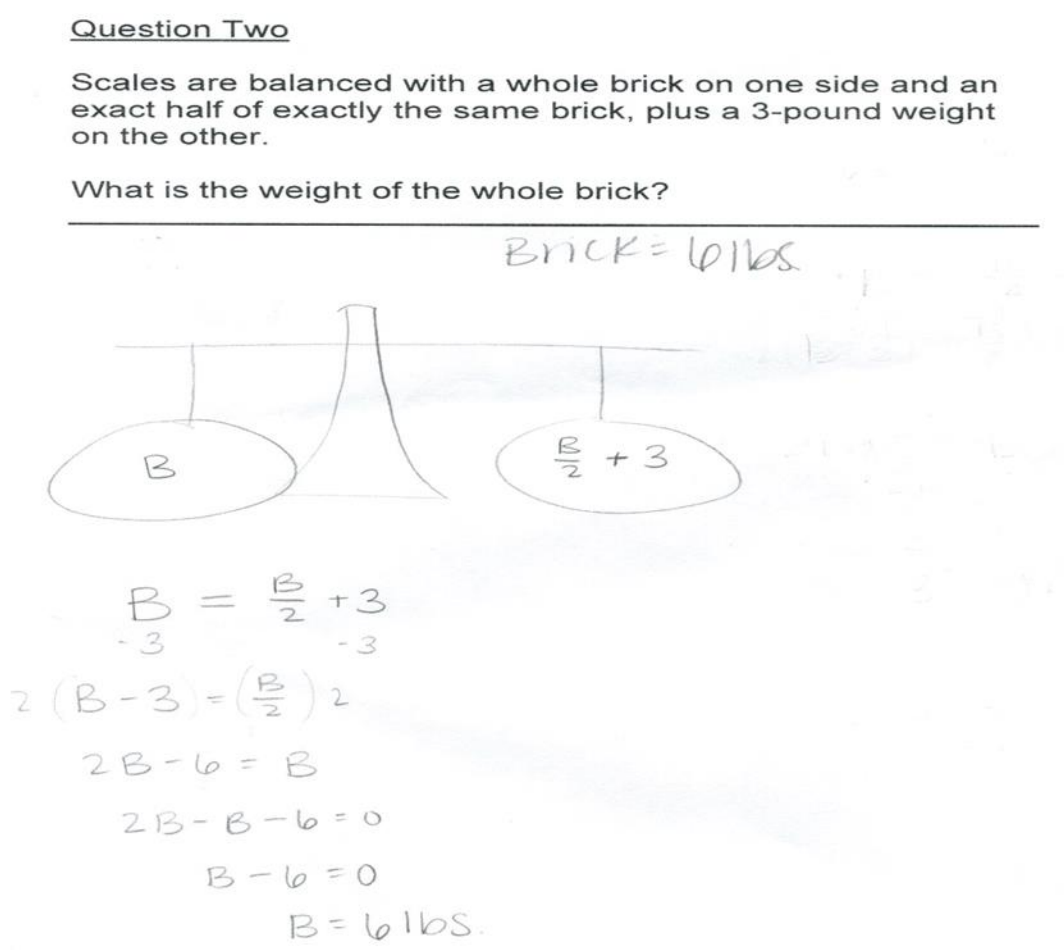


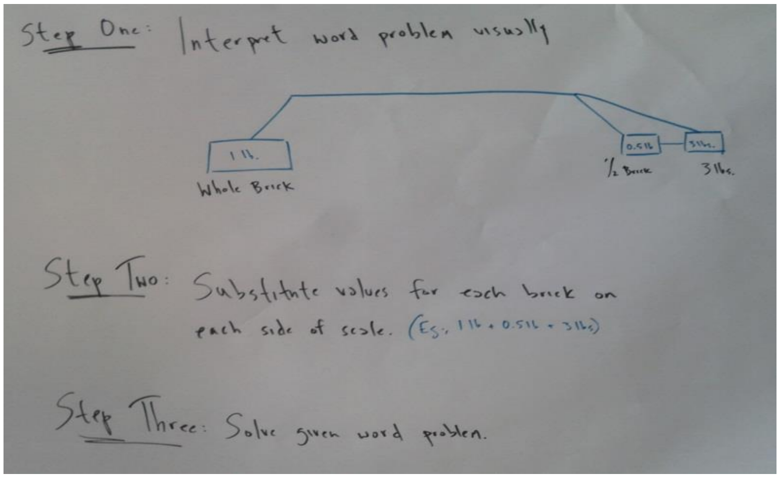
Figure 4. Case Isabella's Initial Calculation

The Visuals

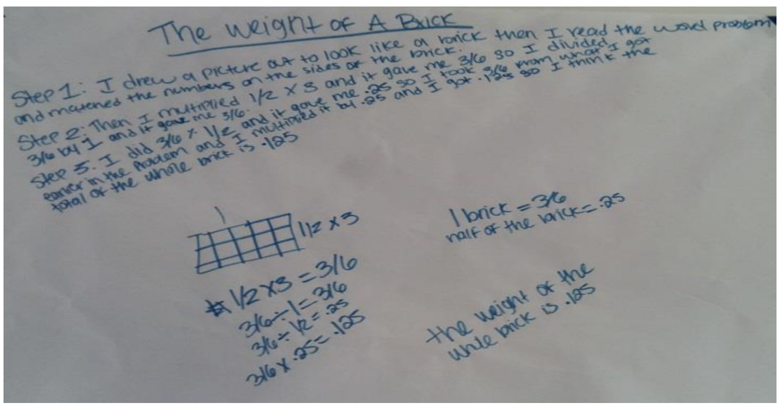
As with the initial calculations, the speech and visuals are presented close together for comparison; as mentioned previously, the visuals are displayed on a webpage at The Brick Problem, password otter17. Each visual represents how participants conceptualized the problem with the goal of teaching the solution to others, in addition to any changes they made from the initial calculations. As with the initial calculations, the visuals are a rich source of data to understand how the participants are thinking about the problem.

Table 1. The Three Visuals for the Video (For larger pictures go to The Brick Problem, password otter17)

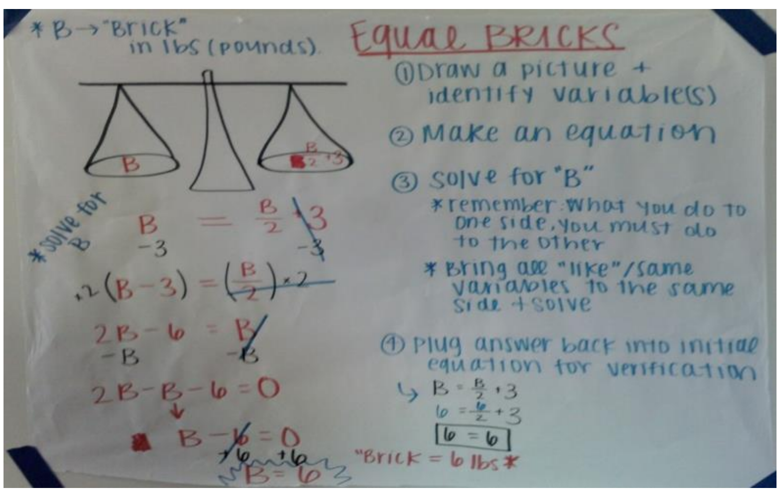
Case Margo's Poster



Case Sandra's Poster



Case Isabella's Poster



Similar to the initial calculations, the portrayal of a scale is the most prominent feature of the visuals. Though limited, the visuals provide insight into the participants' reading comprehension. The question implies that the "scales are balanced," with two sides having the exact same measurements. Margo and Isabella rightly sketch scales with bricks on each side. Sandra does not conceptualize a scale: she draws a brick. Moreover, she writes her explanation more explicitly than Margo and Isabella, and the writing sweeps upwards. Overall, as far as her path from the original problem to solution, Case Sandra does not seem to be conceptualizing two bricks or any kind of balance.

Margo and Isabella both conceptualize a scale, but Isabella is very detailed about each step in the process, in addition to conceptualizing the scale. The way she thinks about the problem and her path from the problem to a solution is clearly presented on her visual. Isabella expands on her initial calculations on the original sheet of paper to the poster paper in a comprehensive manner, presenting each step needed to solve the problem. In contrast to Isabella, Margo does not present much detail on her visual, and Margo excludes much of the initial calculations from the poster she uses to explain the problem.

Speech and the Video Presentation

The speech, transcribed below in Table 2, is *inseparable* from the visual and pointing that occur on the video. As with the initial calculations and visual, we are juxtaposing the speech from each participant. During the video, the researcher asked Sandra to reflect on her presentation to emphasize metacognition. This part of Sandra's speech is presented separately.

Table 2. Transcribed Speech from the Videos (see also The Brick Problem, password otter17) Bolded words for Case Margo mark an explicit metacognitive act. Also, "So" is bolded to highlight a prominent pattern in two of the participants' transcripts.

Case Margo Step one says interpret the word problem visually This is my scale that I've drawn. On the left side I've drawn a whole brick. And, uh, on the other side of the scale, which is supposed to be balanced. I've drawn—I brought to scale on half of the brick on the left side plus the three-pound weight on the right side, and this illustration is supposed to show that balance on both sides **so** we have a picture. Step two is to substitute the values for each brick on the left side and the right side of the scale **so** it will be balanced. And I chose simple numbers, one for my whole brick on my left side, **but I made a mistake because half of one is point five plus three is three point five, And one does not equal three-point five.** But that's the idea and once you figure out what number is on the left equal half of the number on the left plus three Then that is the value--Then you're solving um the weight of the whole brick.

Case Sandra **So** first I did step one I drew a picture to look like a brick; then I read the word problem and matched the numbers on the side of the brick. Well with then the brick what I do actually, I put one up here, and one down right here cause this I want to be two whole sides **So** I put the one there cause this side is by one half and when I read the words on this side, it said add three But I times three—no, uh, but it said two. Then I multiplied one half times three, and it gave me three-sixths **so** I divided. **So** here you go, one half times three Equals three-sixths, **so** I did three sixths divided by one half which shows right—(stick) three sixths divided by one half, and that gave me point twenty-five**. So** I took three-sixths from what I got earlier, which is one half—no I got point two five, that's it (I mean), **so** (I did) three-sixths times point two five, it gave me one twenty-five. **So**, um, basically, I did the weight of the whole brick was point one twenty-five and that's what I wrote the weight of the brick.
Case Isabella: **So** the first thing I did was draw a picture of a scale that showed each side We knew that there was one brick on one side, and the other had half the same—pound of brick on the other side plus three. I put B because B equals a brick in pounds. **So,** the first step was to draw a picture and identify variables or variable. Step two was to make an equation, **so** I put B equals half B plus three. **So** then I have to solve for B. Remember what you do to one side you have to do to the other side, and *to* bring all like or the same variables to the same and solve it. **So,** what I did was subtract three on both sides **so** I got B minus three equals one-half B; multiply that by two Uh—on both sides to get B by itself over here. **So,** I have two B minus six equals B. Then I subtracted B on both sides: ended up with zero. **So,** you can solve with B. So you have two minus B minus six equals zero, which simplifies out to B minus six equals zero. So then I solve for B add six to both sides and B equals six. **So,** the Brick equals six pounds **So** then I plug the answer back into the initial equation; this one for verification. **So** right here I have B equals one-half B plus three; six equals six over two plus three. six equals six Ta Da.

Case Margo

At the beginning of her video presentation, Margo emphasizes the scale, with both sides "balanced." As mentioned previously, she demonstrating some reading comprehension by conceptualizing "scales." She also concludes her explanation with this: "once you figure out what number is on the left plus three; then that is the value. Then you're solving um, the weight of the whole brick" (see The Brick Problem; password otter17).

During the interview and after she watched the video, Margo mentioned how she noticed an error when she moved from the initial calculation to the poster paper:

> "I realized I caught my mistake while I was drawing from, um that paper and not this paper (she is pointing to the poster on the wall). All of a sudden I realized I had done it correctly here, but then really looked at it on there (she is pointing back and forth between the initial calculation and the poster)".

This is an important metacognitive act, though in this example, the metacognitive act is comprised of what she found earlier, moving from the initial calculation sheet to the poster. During her video explanation, she illustrates a new metacognitive act, a new path from problem and solution.

To summarize Case Margo's metacognitive acts, these occurred at each step in the process and can be seen in the data at several points. On her initial video, she is making guesses after noting that some of her numbers are not working out. As she moves from her initial calculations to the poster paper, the stripping down of her original calculations suggests that she has found more problems in her representation of a solution; perhaps she has lost confidence too (i.e., low self-efficacy). Then on the video, she supplies some specific numbers that should have been different, which are still not accurate. The diagram and her explanation on the video suggest the main problem in her solution was assigning the number "one" to the brick on the left side of the balance. Although she mentions this, Margo does not seem to realize that this was not the right choice, but as the second author pointed out, this was a logical choice.

Case Sandra

On the video there is one section where her awareness that she did not make a correct choice indicates metacognition, though not directly. Her confusion about addition and subtraction is indicated by the following in the video data: "I read the words on this side, it said add three, but I times three—no, uh, but it said two. Then I multiplied one-half times three, and it gave me three-sixths, so I divided." When she says that "times three", she does so with a rising tone as if "three times" is a question." She also wiggled the pointer back and forth, which could be interpreted as related to the gesture category called a *beat.* This suggests she might be searching for the right word or concept (see McCafferty, 2002; McNeil, 1992), although the gestures are linked to the visual and the pointer: this is a variance of McCafferty(2002), and McNeil's (1992) perspectives on *spontaneous gesture*. Sandra also mentions the addition and multiplication confusion during the recorded conversations after watching her video.

While still on video after she had completed her presentation, it was evident that she was not confident with her answer, and it seemed sensible at that moment to try to prompt metacognition. This is the conversation that began on the video as soon as Sandra ended her explanation.

> Interviewer: Do you see anything now that might be a weak point? What do you think is a weak point? Anything?
>
> Sandra: Yeah, I feel like right there when I multiplied three, when it said add three, in the word problem, I multiplied.
>
> Interviewer: It should have been addition there?
>
> Sandra: Yeah it should have been
>
> Interviewer: I wonder what caused the confusion. Do you think it's the shape of that? (the interviewer is referring to the brick shape on the visual).
>
> Sandra: No I feel like cause I'm not used to multiplying again so I added. So this one gonna be worse cause I'm used to the word problem when you multiply, and they say add. So um—

Interviewer: that's where the error might be

Sandra: Yeah

Several metacognitive acts occur during this end-of-the conversation section on the video. She has thoroughly assessed a calculation error and repeats it on the video; then returns to this theme of multiplying when she should have been adding. After watching the video and being asked what she would do differently, she replied, "Really it was the problem, because like, I started off wrong because of a new thing, and I multiplied, so this is what threw me off." When asked about improving the whole process, she said that "Reading the question over until you can figure it out, like what you're doing."

To summarize Case Sandra's metacognitive acts, she is aware of her error in multiplying instead of adding on the video and during the post-video conversations. She also noted that reading the question carefully is important, so she is indicating some awareness that she missed the premise of the problem. Sandra also does not conceptualize a balanced scale, and instead conceptualizes one brick into pieces. Around these bad choices and errors in her equations and guesses, she is exhibiting metacognitive acts, but these metacognitive acts were not made explicit until she was asked. Some metacognitive acts appear more indirectly, as when she waves her pointer around while mentioning "multiplied three times." This occurs around other false starts and pauses in the discourse.

Case Isabella

For Isabella the three major modes of speech, the visual, and the act of pointing are coordinated and precise (see The Brick Problem password: otter17). Her speech is smooth and uninterrupted. She integrates the variable B into the concept of balance, packaging the variable and the metaphor of balance together into a vivid image both orally and on the visual. She explicitly reminds the audience of balancing the equation, mentioning the necessary metacognitive act of verifying: "Remember what you do to one side you have to do to the other…So then I plug the answer back into the initial equation; this one for verification." For Isabella, the data suggests that her metacognitive acts are internalized and only mentioned as a way to make it clear to the audience. During the post-video conversation, when Isabella was asked what the hardest part was, she mentioned "the initial set up of the problem, like putting it in terms of like, transferring words into an equation."

Discussion

Our purpose in this ongoing research has been to explore the power and potential of digital video for unpacking students' abilities to solve math word problems; the emphasis is on metacognition, specifically, metacognitive acts. This first paper also presented a theoretical framework with the emphasis on analyzing the synthesis of speech, visuals, and acts of pointing. The paper is also intended to demonstrate how digital video, with speech, visuals, and acts of pointing as units of analysis can be adapted for instruction and assessment. Finally, our study aligns with the trend in the literature for a semiotic approach to the study of math word problems, in addition to calls for a closer examination of the relationship between reading, visuals, interaction, and an emphasis on the negotiation of meaning (Greenberg, Ginzburg, & Wrigley, 2017; Goos et al., 2002; Kong & Orosco, 2016).

Metacognitive Acts

By using speech, visual, and the act of pointing as units of analysis, metacognitive acts can be identified and tracked as a multi-modal *genesis* of negotiated meaning by teachers and students. For example, the data provided insight into how participants were thinking about the problem in a way that can inform their efforts to "edit" their solutions of word problems.

Prominent examples of metacognitive acts from the present study occur across modalities from the initial calculation to the poster, and then finally to the video; this includes short background interviews and participant comments on their problem-solving steps and strategies. One prominent metacognitive act that can be found in the data is when Margo excludes some of her initial calculations from her video-presentation poster (i.e., the visual), which is stark in appearance. With regard to Margo engaging in "…an assessment of a particular aspect of a solution" (Goos et al. p. 199), this is similar to students condensing and/or editing content from one draft to the next in academic writing (see Unger, 2018).

The genesis of another metacognitive act occurs during Margo's video presentation when she points to the scale on her visual and says, "I chose simple numbers, one for my whole brick on my left side, but I made a mistake because half of one is point five plus three is three point five, and one does not equal three-point five." Margo's metacognitive act was primarily self-regulated and was prompted through the process of explaining by using her visual.

In a different manner, Sandra had to be prompted into a metacognitive act by the first author asking her a question about a "weak point" in her calculations. She replied with "yeah, I feel like right there when I multiplied three when it said to add three in the word problem, I multiplied." Asked by the first author if the shape of the brick was a problem, Sandra returned to the problem of multiplying when she should have adding. Overall, Sandra's data suggests that reading comprehension is an issue moving from the two different semiotic systems of the English language and the Math Language.

The pace and content of the speech on video, with the visual and pointing as inseparable cognitive resources, displays a comprehensive view of the participants' understanding of the problem and their belief that they are on the right path from the initial calculation to conclusion. The metacognitive acts already identified in the transcripts include Margo's reflection during her presentation about her calculation. Sandra's difficulty in moving between the semiotic systems of math and English is also salient in the occasional false starts, along with her noticeable rising tone and beats with the pointer around parts of her explanation that were not clear to her. Isabella, in contrast to both Margo and Sandra, was quite clear and confident in her presentation, with her visual coinciding clearly with the steps needed to reach the correct outcome. Most striking about her transcripts was the consistent use of the coordinator "so" to signal a next step. "So" also appears in Sandra's transcript, but not as often and not with the consistency as with Isabella. Sandra does not use "so" only once, and other transitions were not prominent. The precise use of transitions as indicators of metacognitive acts needs more investigation.

Math Word Problems, Reading, Videos, and Visual Displays

In the process of analyzing the data for metacognitive acts, important questions emerged about the relevance of the participants' reading abilities and how much the videos and visuals added to the participants understanding of the math word problem. Although the reading abilities of the participants were not a main objective of the study, as with other studies in our review, we found that reading comprehension and semiotic mediation (i.e., a cycle of interpretation, signification and mediation) are inseparable from the participants success with word problems.

Recall Sandra's striking difference in leaving out the concept of "balance" and "scale": her focus was on the brick and not the balance of both sides of a scale, as compared to Margo and Isabella. Also, she focused on dividing the brick and multiplied instead of adding. She revealed her confusion about the problem in response to the questions from the first author and after reflecting on her video (see Table 1 and the transcripts). As mentioned earlier, Sandra's problem seems to be a reading comprehension issue. Reading comprehension was suggested in Margo's interpretation, though making any direct cause and effect statement is beyond the scope of this study.

Sandra, Margo, and Isabella, through the entire problem-solving process, are interacting in a way that Sáenz-Ludlow's (2006) called "dialogic interaction" (p. 203), a term she used in describing interpretation games. She also states that "Social interactions between teacher and students and among students themselves are *acts of communication* constituted at two levels: acts of communication with *oneself* and acts of communication with *others*" (p. 196). This aligns with the views of Wertsch (1998) and Bakhtin (1986; see also Clark & Holquist, 1984), which also take the view of a dialogic construction of the *self* as other. (Wertsch, 1998, and Baktin, 1986, are also cited by Sáienz-Ludlow, 2006). All three participants are enacting a *self-as-other* view as they place language and image in front of them to create language and image for an imagined audience. Through this process, each participant discovers something about their understanding of the Brick Problem's author-intention.

This self-as-other perspective is important in terms of understanding the power of multi-modal representations of the problem-solving process. For the participants in our study, they were prompted to have an imagined audience (i.e., the emphasis on reciprocal teaching), in addition to having the researcher to interact with, or with some of the participants, they showed up together to work on different problems so they had another participant who was in the vicinity.

Also important to our study is Sáenz-Ludlow's (2006) findings that the interpreting games and language games mediated the students' comprehension of simple math word problems. For our participants, each step in the process, specifically when they were preparing the visual representations and presenting their explanations on video, prompted a self-as-other perspective.

With regards to using visuals and multimedia, our approach, though different, followed along with the suggestions of Casey (2003) and others (Kong & Orosco, 2016; Sáenz-Ludlow, 2006). Each of these authors also used variations of Vygotskian-related ideas that align with our intentions for the visuals, the explanations, and the videos to act as a kind of "instructional scaffolding" (Kong & Orosco, 2016, p. 171), which can support students' comprehension of the word problems. Other scholars who align with our Vygotskian notion of tool use and the powers of visuals and digital tools include Lantz-Andersson, Linderoth. and Säljö (2009). These researchers also used discourse analysis of students collaborating and discussing problems appearing on a computer monitor. As with our participants, the participants in Lanz-Andersson et al. used a digital tool to mediate their responses.

A semiotic-Vygotskian approach to mathematics, along with Tomasello's (2003) understanding of the collaborative and social nature of language acquisition, can provide a productive avenue for understanding how students move back and forth between natural language and the language of mathematics (Lemke, 2003).

Conclusion

Our direction with creating a multi-modal process of mediational means follows along with many other scholars we reviewed. To better involve the reader in judging our descriptions and interpretations, we offer the raw video data at The Brick Problem (password: otter17). In closing, it is worthwhile to quote Sáenz-Ludlow and Presmeg, (2006), "…the teaching and learning of mathematics is essentially a collaborative semiotic activity mediated by the simultaneous use, re-creation, interpretation, and appropriation of a variety of semiotic systems" (p. 2).

Limitations

As with all research, this study has limitations. Because three cases are presented, it is difficult to generalize beyond the immediate contexts. However, this research can be used as a reference to create a variety of classroom-based video activities with speech, a visual, and the act of pointing as accessible units of analysis. With regards to objectivity, because of our deep involvement in positioning the word problem, the students, and the interaction in a specific manner, in addition to the interpretive descriptions, the findings are interwoven with the researchers' perspectives.

Avenues for Further Research

Many avenues for further research can follow along these same general steps and the theoretical approach presented by the current study. Moving the procedures into more of a mixed-methods design with control and experimental groups is an important avenue to find any cause and effect or correlational relationships between the digital video (perhaps the whole set of activities described above) and learning outcomes. Another option is to create a class specifically on word problems from a variety of disciplines and teach students how to study their responses. The options for adjusting these basic steps and overall approach to language teaching are many.

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